LOW THRUST TRAJECTORY DESIGN AND OPTIMIZATION: CASE STUDY OF A LUNAR CUBESAT MISSION

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• ABSTRACT

The trajectory design and analysis for a low-thrust lunar CubeSat mission is presented. The CubeSat starts on an Earth-escape trajectory with the primary goal of attaining stable lunar orbit within 1 year. The nominal trajectory is first designed using impulsive burns, then transitioned to a low-thrust finite-burn model, as is typically done for low-thrust trajectory design. Multiple trajectory constraints are considered, including avoiding repeated Earth flybys, limiting the maximum thrust duration, and minimizing risks associated with failed burns.

Several benefits of using the NASA General Mission Analysis Tool (GMAT) for trajectory design are also demonstrated. Specific analysis using GMAT includes an investigation of the Earth-Moon-Sun dynamics as they apply to the initial lunar-flyby trajectory, the use of the Sun-Earth L1 Lagrange point to transition from an Earth-departure to a lunar-arrival trajectory, and the design of a Lunar Distant Retrograde Orbit (DRO) that provides a safe starting point for the final lunar orbit.

Index Terms— trajectory, optimization, CubeSat, low-thrust, GMAT

1. INTRODUCTION

NASA kicked off its Centennial CubeQuest Challenge in November 2014 in order to encourage non-government organizations to rapidly develop low-cost 6U CubeSats. Competitors that demonstrate a high probability of success over multiple competition stages are selected for a launch on the 2018 Exploration Mission 1 (EM1), which will place their CubeSat on a lunar flyby trajectory. In addition to an end-to-end hardware and control system design, competitors must design a trajectory that will achieve stable lunar orbit and transmit data for one month – all within one year after launch. While larger, more traditional spacecraft could easily accomplish this via direct lunar orbit injection, CubeSats are generally limited in their thrust and control capabilities and therefore require unique and interesting trajectory designs.

This paper presents the design of a nominal trajectory that accounts for both prescribed and self-imposed high-level requirements, and achieves final lunar orbit within 11 months of the epoch. The prescribed requirements, provided by the CubeQuest Challenge rules, set the mission's epoch and initial conditions, duration, and final lunar orbit parameter ranges. The self-imposed trajectory requirements serve to further increase safety margins and allow for contingencies in case the nominal trajectory cannot be achieved (e.g. underperformance of the control or propulsion systems).

The nominal trajectory design is approached in three phases, each of which incorporates additional trajectory requirements. First, the design space of the Earth-Moon-Sun system is explored to glean information on possible trajectory solutions. This results in initial guess trajectories that depart the Earth and arrive at the Moon, which are used as inputs to the second design phase: the impulsive-burn trajectory solution. In this phase, GMAT is used with the VF13AD optimizer to obtain a continuous trajectory using impulsive burns.[1,2] This trajectory is then used as the initial guess for the third design phase, in which the impulsive burns are converted to low-thrust burns and re-optimized using GMAT and VF13AD to create the final nominal Earth-Moon trajectory. It is shown that this trajectory results in a final lunar orbit that is stable for significantly longer than the competition duration.

2. EARTH-MOON-SUN DESIGN SPACE

2.1. Earth Disposal and Lunar Flyby

The CubeQuest Challenge provides the initial conditions, called the disposal state, on which all trajectory design is based.[3] This disposal state is given in Table 1.

Table 1. The CubeQuest disposal state, as provided by the CubeQuest Challenge requirements.

Parameter	**Value**	**Unit**
Epoch	Dec 15, 2017 14:56:42.2	TDB Time
X	-1.501540312811781E+04	km
Y	-2.356897680091111E+04	km
Z	2.241504923500546E+03	km
V_X	-4.855378922082240E-01	km/s
V_Y	-5.048763191594085E+00	km/s
V_Z	-8.799883161637991E-01	km/s

Figure 1 shows that, when propagated using an Earth-Moon-Sun point-mass force model, the spacecraft gains sufficient energy from a trailing-side lunar flyby to escape the Earth-Moon system entirely. Here, the flyby occurs 4.2 days after the initial epoch. In order to prevent this escape, the lunar flyby must be adjusted to drastically reduce the amount of energy gained. Slowing down the CubeSat, thereby allowing the Moon to move away from its flightpath, accomplishes this goal by increasing the flyby distance and consequently decreasing the amount of energy gained during the flyby.

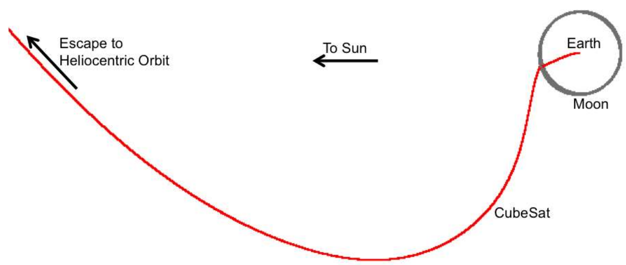

Figure 1. CubeSat disposal trajectory in the rotating Sun-Earth reference frame. Lunar flyby occurs 4.2 days after epoch.

Figures 2 and 3 depict two possible trajectories resulting from applying an impulse of 7.49m/s and 12.65m/s, respectively, in the anti-velocity direction approximately 1 day after the epoch. Both trajectories return to the orbit of the Moon in 150 and 165 days, respectively. It should be noted that these trajectories are very similar to those found by Folta et al.[4]

This preliminary investigation of the Sun-Earth-Moon design space reveals multiple trajectories, which are significantly different from each other, that use small impulses to alter the lunar flyby parameters and eventually return to the Moon. In fact, there are many such trajectories, and choosing between them involves mission-level requirements as discussed further in Section 3.

2.2. Lunar Arrival and Distant Retrograde Orbit

There are multiple methods for establishing lunar orbit after arriving at the Moon, and choosing between them involves addressing mission-level requirements. Given a self-imposed requirement of assuring a wide safety margin in case of propulsion system failure, it is important to use a lunar orbit that stays within the vicinity of the Moon for as long as possible. To this end, the class of lunar Distant Retrograde Orbits (DROs) is of great benefit since their stability is well understood [5-7]. An example of DRO stability is shown in Figure 4, which shows the time history of a DRO that starts 100,000 km from the Moon. It can be seen that this orbit remains in the vicinity of the Moon for more than 2 months, even when propagated with ephemeris models for the Earth, Moon, and Sun. This stability is observed in DROs at even larger lunar distances.

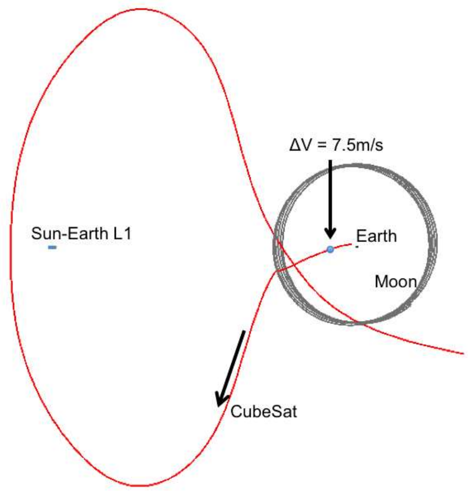

Figure 2. Pre-flyby impulse resulting in direct return to lunar orbit.

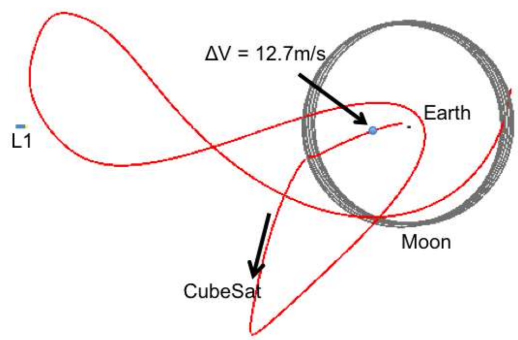

Figure 3. Pre-flyby impulse followed by an Earth flyby, resulting in return to lunar orbit.

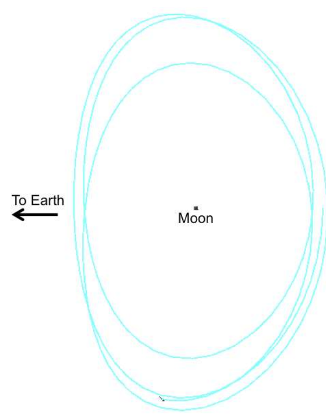

Figure 4. Lunar DRO starting at 100,000 km and remaining in the immediate vicinity of the Moon for 60 days. Trajectory shown in rotating Earth-Moon reference frame.

Furthermore, if a small impulse is applied to the initial condition of the DRO then backward-propagated, the resulting trajectory (which lies on the DRO's stable manifold) will allow a CubeSat to reach the Moon from the vicinity of Sun-Earth L1 with little cost. An example of a backward-propagated trajectory is shown in Fig. 5.

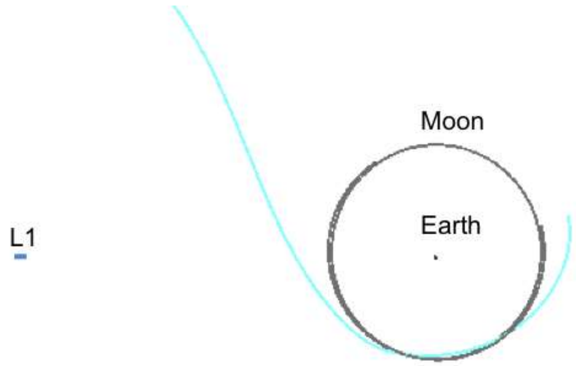

Figure 5. A trajectory on the lunar DRO's stable manifold.

2.3. Sun-Earth L1 Dynamics

When comparing Fig. 5 to Figs. 3 and 4, it becomes apparent that the portions of the Earth-departure trajectories that occur after Sun-Earth L1 could, with proper numerical targeting, be connected to the lunar-arrival trajectory that eventually meets with the lunar DRO. In fact, as shown in Section 3, a properly-placed impulse near the Sun-Earth L1 does allow for a smooth transition from the Earth-departure trajectory to the desired lunar stable manifold.

2.4. Spiral-Down to Lunar Orbit

Starting from the DRO, establishing a final lunar orbit involves applying impulses that successively reduce the orbit's size. This method is quite fault-tolerant, since a missed impulse (e.g. due to thruster failure) will not result in a total divergence of the CubeSat trajectory. However, as explained in Section 4, two downsides of this approach are long time to final lunar orbit and continuous anti-velocity steering as the trajectory nears the final lunar orbit.

2.5. Propagation Force Models

All simulations for this study were performed using high-fidelity force models available in GMAT. All trajectory legs included at least point-mass ephemeris models for Earth, Moon, Sun, and Jupiter (the next most gravitationally significant object in the design space). Additionally, solar radiation pressure was modeled assuming a CubeSat area of 1 m^2, reflectivity coefficient of 1.7, and 14 kg mass. When near the Earth (e.g. immediately after disposal), a 100x100 EGM-96 gravity model was used. When near the Moon (e.g. during spiral-down), a 100x100 LP-165 model was used.

3. IMPULSIVE-BURN SOLUTION

3.1. Requirements Affecting Trajectory Selection

Section 2.1 introduced two types of trajectories that allow the CubeSat to gradually approach lunar orbit: those that utilize Sun-Earth L1 dynamics (Fig. 2) and those that perform Earth flybys (Fig. 3). Choosing between these requires analyzing mission-level requirements. For this study, there is a self-imposed requirement that the CubeSat avoid repeated transits through the Van Allen belts, as well as known satellite constellations (LEO, GEO, etc.). This requirement reduces the risk of system failures due to radiation exposure, as well as risks related to impinging on existing Earth-satellite orbits. Therefore, for this study only trajectories that do not perform Earth flybys are considered; specifically, those of the type in Fig. 2.

3.2. Impulsive Trajectory

The Earth-departure trajectory in Fig. 2, combined with the lunar arrival trajectory in Fig. 5, is shown in

Fig. 6. This trajectory has 3 segments, each connected with impulsive burns. The first segment begins at the pre-flyby impulse and ends near the Sun-Earth L1 point. The third segment starts at the lunar DRO-insertion impulse and backward-propagates for 10 days. The second segment, then, connects these two segments.

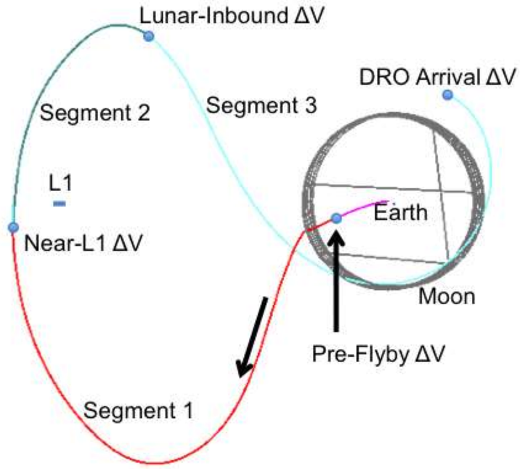

Figure 6. Impulsive-burn trajectory from Earth disposal to arrival at the lunar DRO.

This approach, which fixes the known departure/arrival boundaries and patches an intermediate trajectory, is a type of multiple shooting technique commonly used in trajectory design. Its main benefit over sequentially propagating from the Earth to the Moon is that it avoids convergence issues related to the highly chaotic and sensitive Sun-Earth-Moon system.[8-10].

The first impulse for this trajectory is initialized by visual inspection. Because it is expected that this impulse applies mostly in the anti-velocity direction to slow down the spacecraft, the impulse itself is modeled in the Velocity-Normal-Binormal (VNB) reference frame. GMAT's built-in differential corrector is then used to compute values for the three impulse components that result in matching 3-dimensional position of a point near Sun-Earth L1.

The third impulse of this trajectory puts the satellite into a lunar DRO and is also initialized by visual inspection. In this case, the DRO-arrival impulse is defined in the Earth-Moon rotating frame with zero V_X and V_Z components. The V_Y component is chosen to tangentially depart the DRO (in backwards time) and arrive at a point near Sun-Earth L1 30 days before the DRO trajectory time.

The second impulse is applied at the end of the first segment, near Sun-Earth L1, and the resulting trajectory is propagated to the start time of the lunar-inbound trajectory that was backward-propagated from the third impulse.

Figure 7 shows the process flow that is followed by GMAT to create a continuous impulsive trajectory. There is an "outer" loop, implemented in GMAT using the VF13AD optimizer that computes the optimal times to intercept the near-L1 point and the lunar-arrival trajectory. For any given iteration of intercept times/positions, there are two inner loops that compute the ΔV components (ΔV_X, ΔV_Y, ΔV_Z) to target the desired intercept times/positions using GMAT's built-in differential corrector. Therefore, the entire process can be described as two targeting loops contained within an optimization loop.

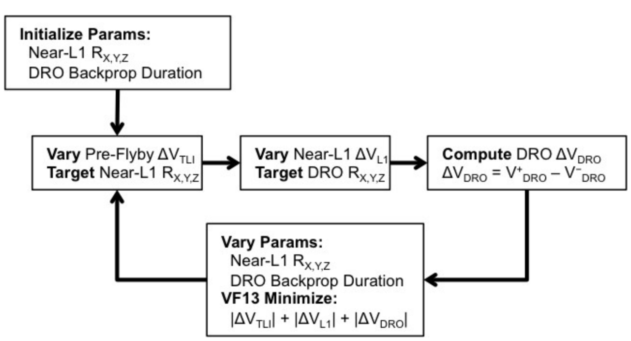

Figure 7. Flow chart to create a continuous impulsive trajectory.

Finally, upon reaching and establishing lunar DRO, the CubeSat begins the process of spiraling-down to the final lunar orbit (Fig.8). The final lunar orbit itself has requirements imposed by the CubeQuest Challenge: apoapse radius less than 10,000 km and periapse altitude greater than 200 km. The spiral-down approach is to coast to lunar periapse, then apply a ΔV of 10m/s in the anti-velocity direction. The magnitude is chosen based on the CubeSat propulsion system, further discussed in Section 4. This coast-impulse sequence is repeated until the CubeSat's lunar apoapse is under 9800km. This extra 200km buffer ensures that secular variations in the apoapse still remain under the 10000 km-ceiling requirement. Additionally, since impulses are performed at periapse, it is guaranteed that only the current lunar apoapse will be reduced. In other words, this spiral-down strategy does not expend any fuel to reduce the orbit size greater than necessary. Therefore, the 200 km periapse altitude requirement is of no concern since the 10,000 km apoapse radius requirement will always be met first.

It should be noted that several parameters for the impulsive-burn solution are chosen by visual inspection. Specifically, this includes the target lunar DRO, the ΔV applied to establish that DRO (which determines the specific lunar-arrival trajectory), and the locations of the spiral-down impulses. While this method is not optimal, the primary goal of developing an impulsive-burn solution is to provide a starting point for the finite-burn solution discussed in Section 4. Therefore, it is sufficient to choose parameters in this manner and use GMAT to create a continuous impulsive-burn trajectory.

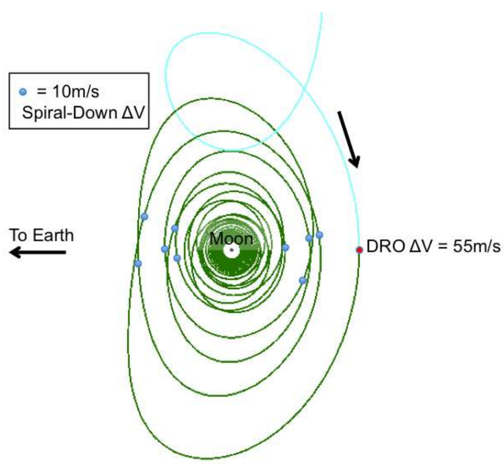

Figure 8. Impulsive lunar spiral-down trajectory, showing first 10 impulses including initial impulse to establish lunar DRO. All impulses (including unmarked ones) occur at lunar periapse.

Table 2 lists the magnitudes of all impulses performed during the impulsive-burn trajectory solution (see Figs. 6 and 8).

Table 2. Breakdown of impulsive-burn trajectory ΔV magnitudes.

Impulse Name	**|ΔV| [m/s]**
Pre-Flyby	10
Near-L1	134
Lunar-Inbound	33
DRO Arrival	55
Spiral-Down (combined)	500
Total ΔV Magnitude	**732**

4. FINITE-BURN SOLUTION

4.1. Impulsive to Finite-Burn Conversion

The impulsive-burn trajectory from Section 3 is used as a starting point to create the nominal finite-burn Earth-Moon transfer trajectory. Per mission-level requirements, the spacecraft thrust and mass . The force-momentum relationship is then used to convert each impulse to the equivalent thrust duration:

The finite-burn maneuver is centered about the impulsive maneuver time, and its direction is set equal to the impulsive maneuver direction and is inertially-fixed over the burn duration. In doing so, another self-imposed requirement must be met; namely, that the initial pre-flyby burn cannot start earlier than 1 day after epoch. This delay allows time for the actual CubeSat disposal, clearing the Van Allen belts, performing hardware and software initialization and testing, acquire a communications lock and obtain several hours of tracking data, perform orbit determination, and re-plan all maneuvers based on the estimated on-orbit state. Details of tracking and orbit determination for this CubeSat mission are provided in a separate paper. [11]

As expected, this conversion is merely an approximation and does not result in a continuous trajectory. However, it does provide a good starting point for further targeting and optimization.

4.2. Optimizing the Finite-Burn Trajectory

The initial-guess finite-burn trajectory is optimized using GMAT and the VF13ad optimizer. The optimization variables include burn durations and directions, specifically:

- Pre-flyby burn: duration, direction
- L1 burn: start time, duration, direction

This constitutes 7 variables since each direction is composed of two scalar angles that define its inertially-fixed unit vector. There are 6 optimization constraints that require continuity in position and velocity at the start of the Lunar-inbound trajectory. The optimization cost function is then the sum of the pre-flyby and L1 burn durations, which is equivalent to minimizing the total ΔV magnitude. Figure 9 shows this trajectory after successful optimization.

Note the lack of a burn that explicitly connects the post-L1 trajectory to the lunar-inbound trajectory, called the “lunar-inbound ΔV” in Fig. 6. This impulse was converted to a finite-burn and added to the optimization setup, but the optimizer determined that the burn duration should be nearly zero. In other words, it is more efficient to simply increase the duration of the L1 burn to directly transfer onto the Lunar-inbound trajectory than it is to insert an additional burn after the L1 burn. Because of this finding, the lunar-inbound burn was removed from the trajectory entirely.

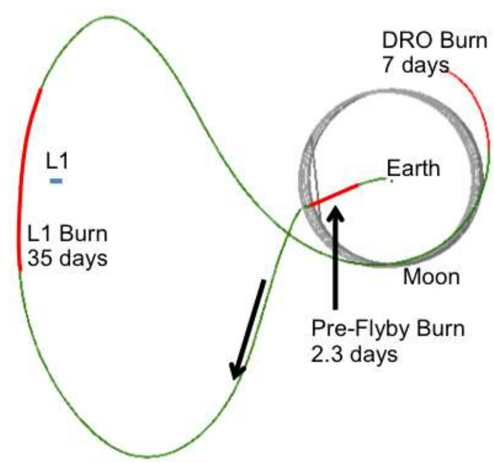

Figure 9. Nominal optimized finite-burn trajectory.

Following a successful DRO burn, the CubeSat must perform multiple burns to achieve final lunar orbit as in the impulsive-burn case. However, because the thrust magnitude is so small and burn durations tend to be long, a new spiral-down strategy was selected instead of repeatedly burning at periapse as in the impulsive-burn case. At the start of the DRO, each burn is computed via VF13ad optimization. The optimization variables are burn duration and direction (3 variables), with one constraint that the following x-axis crossing (in the Earth-Moon rotating frame) occurs purely in the vertical direction, and the optimization cost is thrust duration. The vertical x-crossing constraint takes advantage of the vertical symmetry property of the idealized circular restricted Earth-Moon 3 body system: a trajectory on either half-plane (upper or lower) that crosses the x-axis vertically two times will mirror itself on the other half-plane. This property is often used to generate periodic orbits in similar 3-body systems; here it is used to ensure that the spiral-down trajectory remains in lunar orbit even if the propulsion system fails and a planned burn cannot be executed.

It should be noted that while this strategy of ensuring vertical x-axis crossings results in a very "safe" trajectory, it also increases the amount of time required to establish final lunar orbit. Therefore, it is only used for the first 6 burns, after which the trajectory is close enough to the Moon to be considered in long-term lunar orbit. At this point, the spiral-down burn strategy is changed to more aggressively reduce the orbit size. The thrust duration is fixed at 2 days, and only the inertially-fixed thrust direction is varied. There are no optimization constraints, and the cost function is the periapse radius after the burn. This optimization effectively computes the burn that minimizes the orbit size after each burn, and it is performed repeatedly for 7 consecutive burns. At the end of these burns, the orbit is close enough to the Moon that its period is less than 2 days, and the thrust method is switched from inertially-fixed to velocity-following. The CubeSat thrusts this way continuously until the apoapse radius reaches 9800km.

The lunar-arrival and spiral-down trajectory is shown in Fig. 10. This plot includes the initial spiral-down burns as well as the final continuous-thrust burn, but omits the intermediate 7 spiral-down burns for visual clarity. It is important to note that the long total spiral-down burn duration of 113 days is a classic engineering tradeoff: it is the price paid for having a trajectory that is considerably tolerant to propulsion system failures.

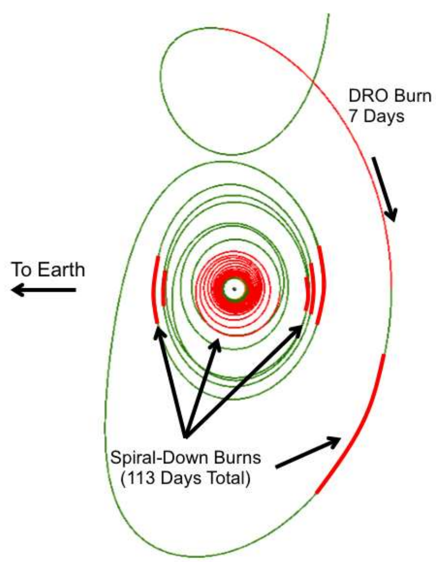

Figure 10. Nominal spiral-down finite-burn trajectory.

Table 3 lists the durations of all burns in the full nominal finite-burn trajectory. Burn durations are converted back to equivalent ΔV magnitudes with the same force-momentum relationship that was used when initializing the finite burns.

Table 3. Breakdown of nominal trajectory finite-burn durations

Burn Name	**Duration [days]**	**|ΔV| [m/s]**
Pre-Flyby	2.3	14
L1	35.0	216
DRO	7.0	43
Spiral-Down (combined)	113.4	700

Total Burn Duration	**157.7**	**973 m/s**
Total Mission Duration	**322.5 (<11mo)**	**--**

4.2. Current Work: Splitting-up the L1 Burn

With a duration of 35 days, the L1 burn is the single longest burn in the nominal trajectory. This poses a risk that, should the burn not be performed correctly, ground control would be unaware of the error until it was too late to correct. Mitigating this risk involves splitting up the L1 burn into multiple shorter burns of approximately 1 week each, with a 1-day coast period between each sub-burn. This day could be used for tracking, orbit determination, and maneuver re-planning. Another benefit of splitting up the L1 burn comes from viewing the trajectory in the Earth-centered J2000 inertial frame, shown in Fig. 11.

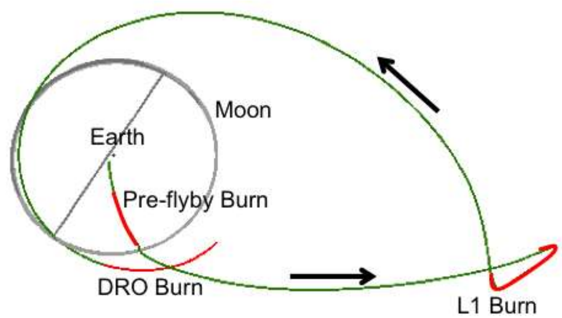

Figure 11. Nominal finite-burn trajectory in an Earth-centered inertial (MJ2000 Equatorial) frame.

When viewed in the inertial frame, it is clear that the L1 burn occurs during a very nonlinear portion of the trajectory as it transitions from Earth-departure to Moon-arrival. Therefore, modeling this burn using an inertially-fixed burn direction appears to be suboptimal. This is in fact true, as preliminary simulations have shown that splitting up this burn and allowing each sub-burn's direction to be separately optimized results in a thrust duration savings of 1.5 days. Additionally, the optimal sub-burn directions are observed to change by over 40 degrees.

5. CONCLUSIONS AND FUTURE WORK

The detailed trajectory design and optimization process for a lunar CubeSat mission was shown, with consideration of requirements imposed by the CubeQuest Challenge and other mission subsystems, as well as self-imposed requirements. The NASA GMAT mission-design tool was heavily used for all simulation, and its built-in differential correction and support for the VF13AD optimizer made it very straightforward to perform all analysis.

One notable issue with the nominal finite-burn trajectory presented in this study is its long total duration and long spiral-down burn duration. Because the CubeQuest Challenge ends 12 months after the CubeSat disposal, it is desirable to reach lunar orbit as early as possible. Adjusting the lunar DRO could allow for a faster spiral-down process and also reduce the total thrust duration for the L1 burn. Additionally, investigating different spiral-down thrust profiles could reveal alternative methods that maintain a fault-tolerant trajectory while reducing thrust durations. Both of these methods will be incorporated into future work.

Other future work includes a thorough analysis of the total ΔV budget, specifically components related to thrust errors. These include errors in the thrust pointing control system and thrust magnitude, as well as errors in solar radiation pressure modeling.

10. REFERENCES

[1] The General Mission Analysis Tool (GMAT), NASA Goddard Space Flight Center, Release R2015a, Nov 2015. http://www.gmatcentral.org

[2] VF13 v1.0.0, The Harwell Subroutine Library, Feb 2011.
http://www.hsl.rl.ac.uk/archive/specs/vf13.pdf

[3] "SLS Secondary Payload Trajectory," NASA CubeQuest Challenge Reference Materials, Dec 17, 2014.
http://www.nasa.gov/sites/default/files/sls_secondary_payload_trajectory_20141017rev1.docx

[4] Folta, D., Dichmann, D., Clark, P., Haapala, A., and Howell, K., "LunarCube Transfer Trajectory Options," AAS/AIAA Space Flight Mechanics Meeting, Williamsburg, Virginia, January 11-15, 2015.

[5] Capdevila, L., Guzzetti, D., and Howell, K., "Various Transfer Options from Earth into Distant Retrograde Orbits in the Vicinity of the Moon," AAS/AIAA Space Flight Mechanics Meeting, Santa Fe, New Mexico, January 26-30, 2014.

[6] M. H´enon, "Numerical Exploration of the Restricted Problem. V. Hill's Case: Periodic Orbits and Their Stability," Astronomy and Astrophysics, Vol. 1, 1969, pp. 223–238.

[7] B. Muirhead, "Asteroid Redirect Robotic Mission (ARRM) Reference Mission Concept Study Public Information Package V1," Aug. 2013. [Retrieved Mar 2016] https://www.nasa.gov/sites/default/files/files/Asteroid-Redirect-Robotic-Mission-Muirhead-TAGGED2.pdf

[8] H. Keller, Numerical Solution of Two Point Boundary Value Problems. Society for Industrial and Applied Mathematics, Philadelphia, 1976.

[9] G. G´omez, J. J. Masdemont, and C. Sim´o, "Quasihalo Orbits Associated with Libration Points," Journal of the Astronautical Sciences, Vol. 46, No. 2, 1998, pp. 135–176.

[10] D. Grebow, "Trajectory Design in the Earth-Moon System and Lunar South Pole Coverage," Ph.D. Dissertation, Purdue University, West Lafayette, Indiana, 2010.

[11] Hur-Diaz, S. and Mathur, R., "Assessing Orbit Determination Requirement with Unscented Transformation: Case Study of a Lunar CubeSat Mission," ICATT 2016, Darmstadt, Germany, March 13-17, 2016.