Use of Hierarchical Mixtures of Experts to Detect Resident Space Object Attitude

David Gaylor
University of Arizona
Jessica Anderson
Emergent Space Technologies, Inc.

Space Situational Awareness (SSA) involves detecting, tracking, identifying and characterizing resident space objects (RSOs). Electro-optical measurements can be used to estimate important characteristics, such as the size, shape, configuration, rotational dynamics (attitude and angular velocity), and surface properties such as specular and diffuse albedo (reflectivity) of a resident space object (RSO). In addition, estimated features can be used to match or discriminate objects or classes of objects and to identify their behavior. Under a Phase I SBIR sponsored by Air Force Research Laboratory (AFRL), Emergent Space Technologies, Inc. investigated the use of a Hierarchical Mixture of Experts (HME) to process electro-optical measurements to determine an RSO's attitude profile. This paper discusses the mathematical background of the HME; the assumptions, test scenarios, and results of processing simulated apparent magnitude and angles data including experiments to tune the HME learning rate parameter. The results show that the HME is capable of identifying and distinguishing between nadir-pointing, sun-pointing, and spinning objects even though none of the experts in the HME is directly estimating attitude. This paper also shows how the learning parameter selection impacts HME performance.

1. BACKGROUND

Space situational awareness (SSA) involves gaining and maintaining knowledge of objects in Earth orbit, particularly objects in the vicinity of valuable space assets. Hazards to space assets are increasing, largely due to the growing number of resident space objects (RSOs) as well as the growing capabilities of potential adversaries. How to effectively automate, schedule, and manage the decision-making associated with maintaining the U.S. Space Object Catalog is a challenging problem of increasing importance to our national security [1]. The U.S. Space Object Catalog currently lists approximately 15,000 trackable objects accounting for approximately 5,800 tons of on-orbit mass. The total population is thought to exceed 20,000 objects larger than 10 cm [2]. Before objects can be catalogued, they must first be characterized. It is necessary to know the location and, preferably, the identity of these objects.

Distant RSOs of interest may be beyond the resolving capability of current electro-optical sensors. Therefore, RSO feature identification must rely on radiometric, spectral, and polarimetric measurements over time. In general, ground-based telescopes with electro-optical sensors are able to provide three measurements of celestial objects: apparent magnitude, right ascension, and declination. The apparent magnitude of a celestial body is a measure of its brightness as seen by an observer on Earth, corrected to the value it would have in the absence of the atmosphere. Right ascension and declination measurements can be used with traditional methods in order to determine the Cartesian position and velocity or orbital elements of an RSO [3]. Apparent magnitude acts as a weak range measurement and also contains some attitude information, so it is able to provide more RSO state information than only angle measurements can provide.

While accurate knowledge of RSO orbital elements is important, to achieve true SSA we must know much more. We must discern the identity, behavior, health, and ultimately, the intent of the RSO. Only then can we differentiate potentially hostile satellites from their benign neighbors. Electro-optical measurements can be used to estimate important characteristics, such as the size, shape, configuration, rotational dynamics (attitude and angular velocity), and surface properties such as specular and diffuse albedo (reflectivity) of an RSO. In addition, estimated features can be used to match or discriminate with known objects or classes of objects and to identify their behavior.

The U.S. Joint Chiefs of Staff set a goal to enhance U.S. Department of Defense (DoD) effectiveness by constructing a system capable of real-time, persistent tracking of RSOs, augmented by advanced sensor fusion techniques capable of inferring probabilistic intent and detecting dynamic change [4]. As part of meeting this goal, the U.S. Air Force is seeking to develop innovative feature estimation or matching approaches to process measurements produced by new and/or existing electro-optical systems.

Under a Phase I SBIR sponsored by Air Force Research Laboratory (AFRL), Emergent Space Technologies, Inc. (Emergent) investigated the use of Hierarchical Mixtures of Experts (HMEs) to process simulated electro-optical measurements of RSOs to determine their attitude profile from three commonly used attitude profiles: spinning, nadir pointing or sun pointing. The electro-optical measurements included photometric measurements (apparent magnitude) and angles (right ascension and declination).

2. HME ARCHITECTURE

An HME is comprised of a number of experts (estimation filters, in our algorithm), arranged in banks regulated by gating networks. The HME algorithm is described in detail in [5]. The HME can be used to estimate the state of an object, even when some of the characteristics or forces acting on that object are unknown. Each of the experts in the HME processes the same measurements, but uses different dynamics and/or measurement models in order to predict future measurements. The residuals between the processed measurements and the predicted measurements are used, along with the associated innovation covariance to assign weights to each of the experts and banks using gating networks. Higher weights are given to the banks and experts that better predict the incoming measurements. A diagram of the HME arrangement is shown in Fig. 1.

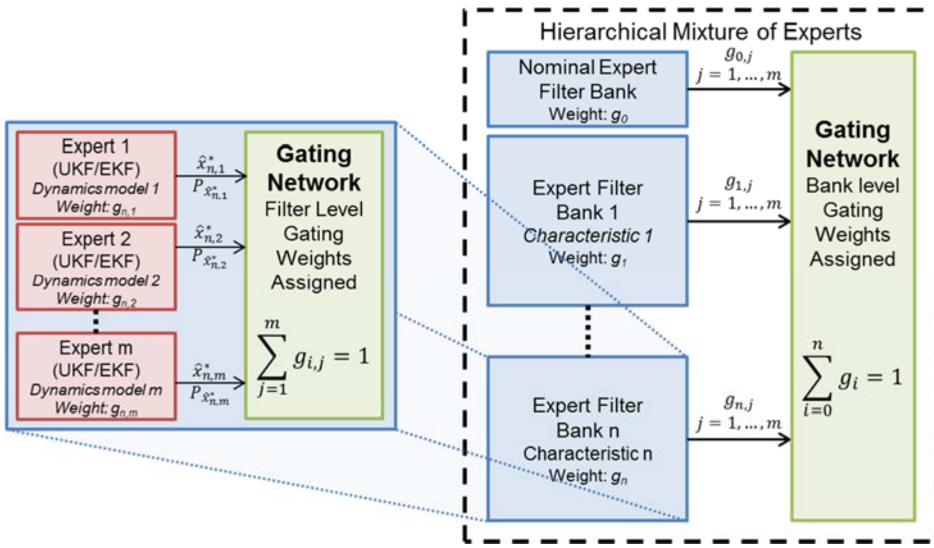

Fig. 1. Two Layer Hierarchical Mixture of Experts

On each layer, a gating network (that is, a simple neural network) acts on the outputs of the experts or banks of experts. The gating network assigns weights to individual filter estimates (or to banks of filters) based on how well the filters predict incoming measurements. A confidence measure, used in adjusting the gating weights for each expert and bank of experts, is provided by the state error covariance. In addition, gating network weights provide a scalar confidence measure that can be used to identify which experts (and, by extension, which dynamic models) best match reality.

Each expert has a slightly different model of the object's state dynamics. All of the experts process the same measurements, but each will produce a different estimate of the object's state. For the purposes of this research, all of the experts used were extended Kalman filters (EKFs). Each EKF uses a different realization of some unknown system parameters, such as size or shape. In the general case, the unknown parameter vector can include different process models as well as different process and measurement noise. Use of higher order filters or other modern filters such as unscented Kalman filters (UKFs) would be possible, and could allow for improved performance over an HME using EKFs. In theory, any type of filter or state estimator could be used as an expert in an HME, as long as it could provide measurement residuals and covariances for those residuals.

The experts and banks of experts in an HME can be arranged to detect micro- and macro-modes in the true state dynamics. In this research, each filter contained a different instance of a micro-mode and each bank of filters modeled a different instance of a macro-mode. An HME can also contain a nominal filter or bank. Nominal filters can help with the detection of RSO features or hinder it, depending on the characteristics of interest. Examples of macro-modes that could be modeled include a change in RSO reflectivity, a change in size, attitude profile, or the existence of measurement noise. Micro-modes are different realizations of the macro-modes, such as individual RSO size models, or specific values of surface reflectivity or measurement noise. For the work described in this paper, the top level banks each contained filters modeling a different attitude profile (major-axis spinning, sun-pointing, or nadir-pointing) and the lower level experts modeled different spin rates, at least in the bank modeling spinning RSOs.

A recursive weighting function (i.e., the gating function) is then used to assign weights (the g's in Fig.1) to the outputs of the filters. Higher weights correspond to the experts in the HME that most accurately represent the true environment. The weight factors are computed as measurements and are processed by the filter bank. The bank of filters learns which filter is performing best by examining a given performance measure, such as measurement residuals and covariances corresponding to those residuals. When all input processing is completed, the highest weight factor will correspond to the best performing filter.

The filter bank update corresponds to intelligently speculating on different realizations of the unknown parameters. The gating network weights are a natural metric to identify groups or classes of events along with confidence in classification assessment. For example, if one of the filter banks contains filters modeling various sizes of cube-shaped objects, and if that filter bank has a high weight relative to the other filter banks modeling other shapes, then this indicates that the object is probably cube-shaped. Examining the filters within that particular bank, the filter with the highest weight indicates the probable size of the object. Filter banks can also be realized for RSOs undergoing component articulation, orientation changes, and other characteristics that can be detected and classified. The mixture-of-experts can classify events which are otherwise indistinguishable, but are indirectly detectable in the tracking data through their effects on the trajectory and observable via electro-optical measurements. For example, a non-continuous property of an RSO (like shape, configuration or attitude profile) cannot be estimated directly in a filter like an EKF, but it can be determined using an HME.

One key feature of the HME framework is that it does not assume that the optimal filter is included in the bank. This lack of complete knowledge is important in this application because we know that practical issues (lack of measurements, non-Gaussian random noise on the measurements, etc.) dominate the estimation error accuracies, and the optimal filter will most likely not be in the bank. The block on the right in Fig. 1 illustrates the overall adaptive filtering structure consisting of a filter bank and a gating network in a forward loop, together with a search algorithm in a feedback loop (not included in Fig.1). The forward loop can be viewed as a multiple hypothesis estimation algorithm, wherein different realizations of the unknown (or uncertain) system parameters are coded into each individual filter in the bank, and the learning network decides which realization provides the best estimate given the available input data. The feedback loop, which contains the search algorithm, is used to periodically update the various filters in the filter bank utilizing the information learned about the system in the forward loop.

The HME uses a single learning parameter for each layer of the HME in order to control its speed and sensitivity. This value can be thought of as a tuning parameter. As it increases, the HME responds more quickly to changes in the incoming measurements, but the HME becomes less able to detect small changes. It is possible for a different value of the learning parameter to be set for each layer of an HME, or to be the same for all layers, depending on the application. The selection of this parameter in our HME algorithm is currently a matter of trial and error, although it has been observed that using a single value between 1 and 3 for both layers of the HME offers good results for most of the cases studied.

3. SIMULATION CONFIGURATION

In order to test the ability of an HME to characterize RSOs, a 6 degrees-of-freedom (6DOF) simulation was used. The simulator models the translational and rotational motion of RSOs, including effects due to the non-uniformity of Earth's gravity field, third-body perturbations, solar radiation pressure (SRP), gravity-gradient torques and atmospheric drag. In order to compute SRP and drag perturbations, each RSO configuration used in the simulation was modeled as a convex system of flat plates. The forces on each plate were computed individually and summed to compute the total acceleration and torque on the RSO. Several different shapes of RSOs can be simulated in the measurement generator, including flat plates, cuboids, and hexagonal prisms (which are meant to approximate cylinders).

The realistic RSO state modeled in the simulation can then be used to generate high fidelity measurements of an RSO, including apparent magnitude, right ascension, and declination.

The apparent magnitude of an RSO, as measured by an observer on the Earth, is a function of the amount of radiant flux received by the RSO from the Sun and of the fraction of light that is reflected in the direction of the observer. This fraction is computed by summing the amount of light reflected by each of the n flat plates that form the body of the RSO model. In order to compute the amount of light reflected by each plate, the Bidirectional Reflectance

Distribution Function (BRDF) is used. The BRDF for an object models the amount of incident light which is reflected by an object, and is defined as the sum of the specular and diffuse reflections, which are both functions of its material properties, as well as the angles of incidence and reflection of incoming light. Thus, RSO apparent magnitude is highly dependent on shape and attitude.

During a previous Air Force Phase 1 SBIR, we made improvements to an apparent magnitude measurement model described by Linares, et al. [6]. These improvements included correcting an observability condition, correcting the amount of reflected light reaching the observer, replacing the specular reflectance term in the BRDF, computing an analytic Jacobian of the measurement for use in an EKF, and modifying the implementation of the BRDF to handle "glints."

In the original Linares model, the apparent magnitude is computed for each surface of the RSO and the value corresponding to the brightest magnitude is accepted as the magnitude measurement. This is valid if and only if one surface of the RSO is illuminated and observable. However, if more than one side of the object is illuminated and observable, this model would be inadequate. In order to make the measurement model truly applicable to a wide variety of objects, both resolved and unresolved, the model was modified to include the contributions of all illuminated and observable reflecting surfaces.

The original model employed the BRDF developed by Ashikhmin and Shirley [7]. However, the highly nonlinear nature of the specular reflectance term contained in this particular BRDF made it less tractable for use in an EKF because the EKF requires the first order derivative or Jacobian of the measurement with respect to the estimated states. A modified version of the original BRDF was found in an unpublished paper by Ashikhmin and Premoze [9], where it was shown that the modified BRDF produced a better overall match to real data. More importantly, it was observed that the spectral reflectance term in this modified BRDF model is of a form that is more amenable to computation of the analytic Jacobian of the measurement. For these reasons, the BRDF was replaced with the modified version described in [8], and the analytic Jacobian of the apparent magnitude measurement model was derived and validated.

Our simulation includes the capability to model three different possible attitude profiles for the RSOs modeled. These attitude profiles are major-axis spinning, nadir-pointing, and sun-pointing. These profiles are modeled by calculating a small moment on the space craft at each time step, which is then propagated by the 6DOF propagation tools to force the RSO model to point in the direction indicated by the selected attitude profile.

4. EXPERIMENT CONFIGURATION

For the experimental results given below, a 100 kg RSO, in a near-geosynchronous orbit,

Table 1. RSO Orbital Elements

a	41000.0 km
e	0.3
i	7.0°
Ω	212.8°
ω	0.0°
θ	282.0°

There were six ground stations modeled in the simulation, although measurements were primarily available from the station in Chile. The latitude, longitude and altitude of these ground stations are given in Table 2.

Table 2. Available ground stations in the HME simulation

Ground Station	Latitude	Longitude	Altitude
Chile	-30.6908°	289.1937°	2220.00 m
Tenerife	28.1887°	-17.5864°	2400.00 m
Maui	20.7081°	-156.2575°	3060.74 m
Texas	30.6714°	-104.0225°	2070.00 m
South Africa	-32.3783°	20.8105°	1798.00 m
Australia	-31.2270°	149.0673°	1164.00 m

All ground stations had a field of view of 50°. The noise on the measurements received by all ground stations had a 1-sigma strength of 1 arcsec for right ascension and declination and 0.0067 for apparent magnitude. For each experiment run, the measurements were available at intervals of 15 seconds to 2 minutes. The RSO was observed for between 1 hour and 8 hours. All experiments were performed with the HME learning parameter set to 1.75 for both the top level and filter level neural networks, unless otherwise noted. This parameter was determined experimentally.

The procedure to run the HME algorithm is as follows. First, initial conditions and parameters for a truth RSO model and for a number of different experts are defined. These initial conditions include a model of the size, shape, surface reflectivity and initial position, velocity, and attitude profile of the RSO for each expert and the truth model. In addition, the parameters for the ground stations (including sensor noise), parameters for the experts (e.g. filter process noise, gravity model specifications, etc.) and any other pertinent and necessary inputs are defined. Then, simulated measurements of the truth RSO are generated using realistic sensor models for the entire length of the observation window. The measurements generated are then sent to the desired number of experts for the HME. For all of the experiments in this paper, all of the experts used were EKFs. The experts use the simulated measurements to generate the inputs for the HME algorithm, which, in this case, are the EKF measurement residuals and their associated measurement covariances.

Finally, the HME processes the measurement residuals and covariances in order to assign gating weights to each expert and bank of experts at each time that measurements are available. These gating weights indicate how well the estimates from each expert agree with the truth model. That is, a higher gating weight means that an expert is more likely to give the "correct" estimate.

The current version of our HME has the following features:

- The code is designed to post-process residuals and innovation covariances from *m* experts in a serial fashion and assumes that all experts processed the same measurements.
- The configuration of the experts into groupings or banks at the top level of the HME is arbitrary and may be configured after data recording. This capability allows for evaluation of macro-mode model change options to be processed after expert data has been generated.
- The current implementation assumes that measurements are uncorrelated for simplicity, but the processing may be modified to consider an innovations covariance with correlation between measurement residuals.
- A single learning rate parameter is used for both layers of the HME, and the learning within each bank is additionally adjusted by the a posteriori probability that that bank is the best fit for the input data stream.

The results of five experiments are included in this paper. These experiments include discriminating whether an RSO is spinning or nadir pointing and detecting the correct spin rate for a spinning RSO, , discriminating between a nadir or sun pointing RSO, detecting an attitude profile change from nadir-pointing to spinning, and detecting a change in spin rate.

5. RESULTS

5.1 Attitude Profile Detection Experiments

Nadir-Pointing vs Spinning RSO Detection Experiment

The purpose of this experiment was to determine if the HME could discriminate whether an RSO was spinning or nadir pointing and detect the correct spin rate for a spinning RSO. The HME configuration for this experiment, shown in Table 3, included one bank modeling nadir-pointing RSOs, containing a single expert and one bank modeling major-axis spinning RSOs, which contained experts using different spin rates for the RSO. The truth model for this experiment was a hexagonal prism with 2m x 4m sides which was spinning at a constant rate (0.3 rad/s) about the major axis. The learning parameter for both layers of the HME was 1.75. The top level gating weights (top) and the filter level gating weights (bottom) for a spinning hexagonal prism RSO truth model indicate that the HME determined the RSO was spinning and selected the correct spin rate.

Table 3. Configuration of HME to Detect Nadir-Pointing or Spinning RSOs

Bank	Attitude Profile	Expert	Spin Rate
1	Nadir-Pointing	1	N/A
2	Major Axis Spin	1	(0, 0, 0) rad/s
		2	(0, 0, 0.3) rad/s

Fig. 2 shows the top and filter level gating weights for the attitude detection HME. The top level gating weights (top) and the filter level gating weights (bottom) for a spinning hexagonal prism RSO truth model indicate that the HME successfully determined the RSO was spinning and selected the correct spin rate. In both cases, an observation window of 1 hour was used, with measurements every 15 seconds. The top level gating weights for the case with the spinning truth RSO show that the highest weight is immediately given to correct bank, modeling major-axis spinning RSOs. The highest weight remains with this bank throughout the simulation. The highest expert-level weight is also immediately given to the expert modeling an RSO with the correct spin rate of 0.3 rad/s. This expert is given the highest weight throughout the simulation.

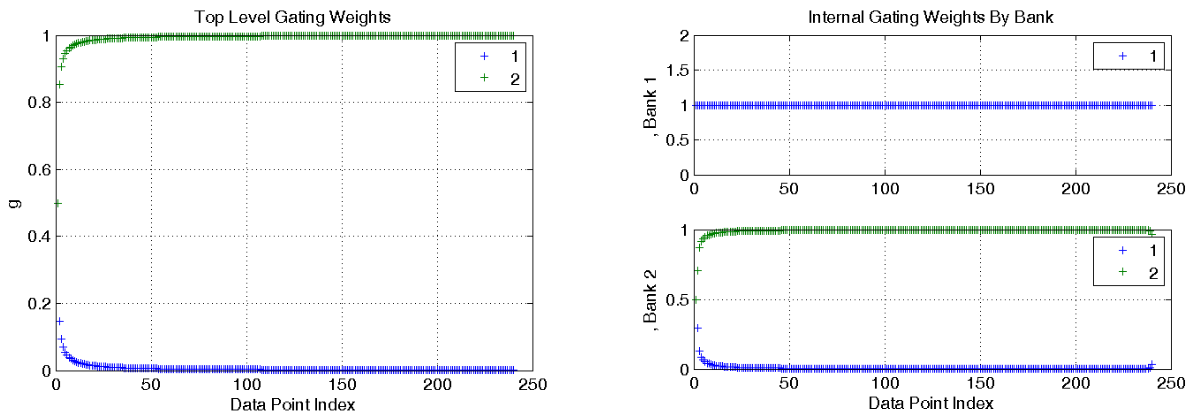

Fig. 2. Gating Weights for the Nadir-Pointing vs Spinning RSO Detection Experiment

Unmodeled True Spin Rate Experiment

We performed an experiment where the correct spin rate was not modeled by any of the filters in the HME to see how the HME would perform in this case. Instead, the spin rate of the modeled RSO was between two of the values modeled in the HME. Ideally, the HME would split the highest gating weights between those filters modeling values on either side of the "truth" spin rate. The configuration of the HME is given in Table 4. For this experiment, the truth RSO is a 2m x 1m x 3m cuboid spinning about its major axis with a rate of 0.5 rad/s.

Table 4. HME Configuration for Unmodeled True Spin Rate Experiment

Bank	Attitude Profile	Expert	Spin Rate
1	Major Axis Spin	1	(0, 0, 0.3) rad/s
		2	(0, 0, 0.7) rad/s
2	Inertially Pointed	1	(0, 0, 0) rad/s

The gating weights for this case are shown in Fig. 3. The HME was able to identify that the RSO was spinning and assigned almost identical weights to the two banks that modeled RSOs with spin rates of 0.2 rad/s less and more than the spin rate of the truth model. It is not expected that the HME would assign exactly equal weights due to the exponential Soft Max function used within the HME algorithm to calculate the gating weights.

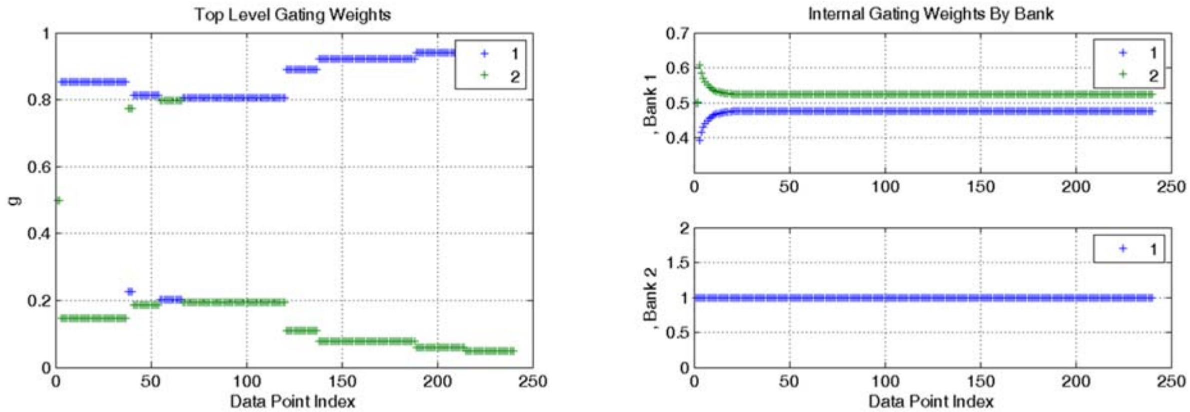

Fig. 3. Gating Weights for the Unmodeled True Spin Rate Experiment

Nadir-Pointing vs Sun-Pointing RSO Detection Experiment

The purpose of this experiment was to determine if the HME could discriminate whether an RSO was nadir or sun pointing. The configuration of the HME is given in Table 5. The truth model for this experiment was a nadir-pointing hexagonal prism with 2m x 4m sides. The learning parameter for both layers of the HME is 1.75.

Table 5. Configuration HME to detect nadir-pointing and sun-pointing RSOs

Bank	Attitude Profile	Expert	Spin Rate
1	Nadir-Pointing	1	N/A
2	Sun-Pointing	1	N/A

Fig. 4 shows the top and filter level gating weights for the attitude detection HME. For a nadir-pointing, hexagonal prism RSO, the top level (top) and filter level (bottom) gating weights indicate that the HME selected the correct nadir-pointing bank most of the time. In both cases, an observation window of 1 hour was used, with measurements every 15 seconds. For most of the second experiment, the highest weight is given to the bank modeling nadir-pointing RSOs. However, the highest weight is sometimes assigned to the major-axis spin bank in error. While the highest weight is incorrectly given to the major axis spin bank, the weights assigned to filters in that bank are temporarily reassigned.

The incorrect bank selection seen in this experiment is likely due to the radial symmetry about the major axis in the hexagonal prism model. In similar experiments with a nadir-pointing 2m x 1m x 3m cuboid RSO and a nadir-pointing 2m x 2m x 2m cube RSO, the HME was more likely to detect that the RSO was nadir-pointing for the cuboid, and less likely to detect that the cube was nadir-pointing. This supports the idea that a high degree of symmetry negatively affects attitude detection. The top level gating weights for the nadir-pointing cuboid and the nadir pointing cube are given in Figure 20. Top level gating weights for a nadir-pointing 2m x 1m x 3m cuboid (left) and a nadir-pointing 2m x 2m x 2m cube (right) indicate that the HME performs better for the less symmetric cuboid than for the cube. In both cases, an observation window of 1 hour was used, with measurements every 15 seconds.

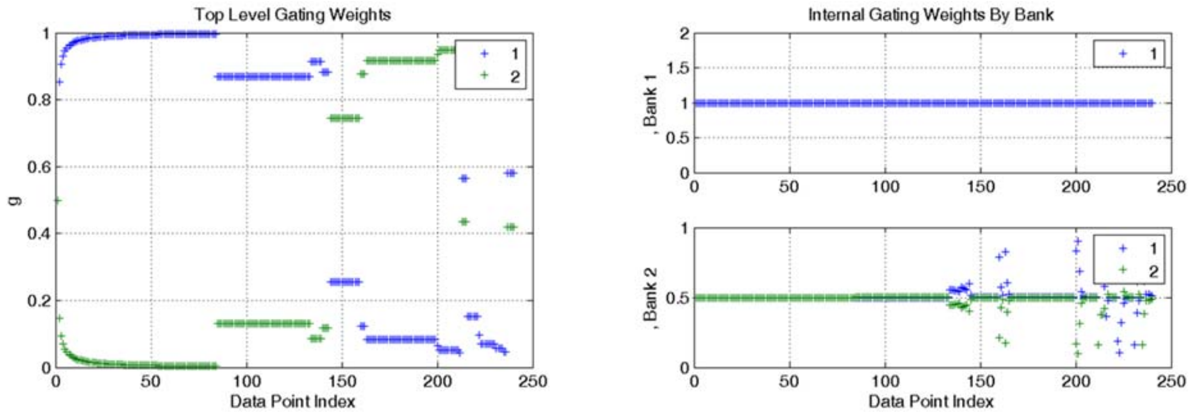

Fig. 4. Gating Weights for the Nadir-Pointing vs Sun-Pointing RSO Detection Experiment

5.2 Attitude Profile Change Detection Experiments

Nadir-Pointing to Spinning Attitude Profile Change Detection Experiment

In addition to investigating whether an HME could identify the initial attitude profile of an RSO, we also investigated whether an HME could detect a change in the attitude profile of an RSO. In one experiment, the RSO being tracked started out with a nadir-pointing attitude profile and then switched to a spinning attitude profile with a spin rate of 0.3 rad/s about the major axis halfway through the simulation. The "truth" RSO model used for this experiment was a 3m x 2m x 1m cuboid. The gating weights for this experiment are shown in Fig. 5 and the configuration of the HME is given in Table 6. For this experiment, the RSO was observed for 1 hour, with measurements available every 3 seconds.

Table 6. HME Configuration to detect attitude profile change

Bank	Attitude Profile	Expert	Spin Rate
1	Sun-Pointing	1	N/A
2	Nadir-Pointing	1	N/A
3	Major Axis Spin	1	(0,0,0.3) rad/s
		2	(0,0,0.5) rad/s

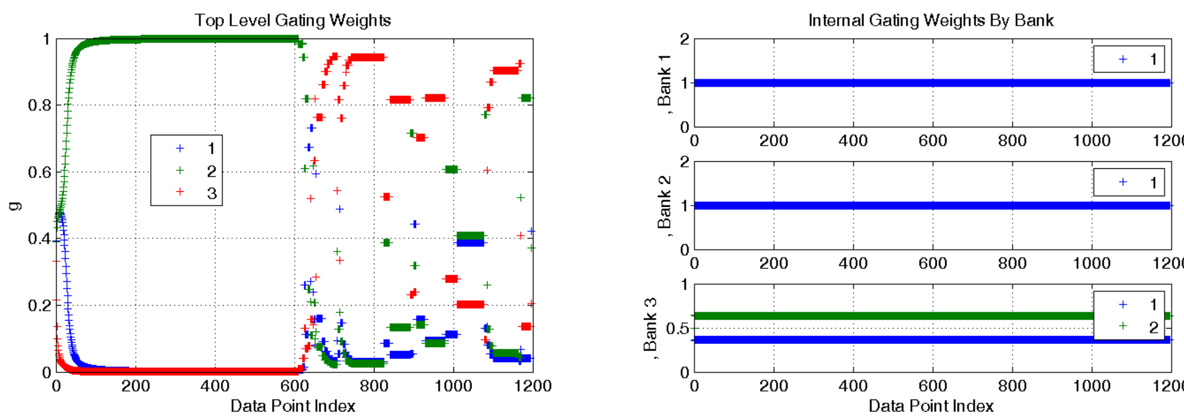

Fig. 5 Gating Weights for the Nadir to Spinning Attitude Profile Experiment

The HME quickly assigned the highest weight to Bank 2, which was modeling a nadir-pointing attitude profile. After the halfway point of the simulation, when the attitude profile of the RSO is changed to a major-axis spinning profile, with a spin rate of 0.3 rad/s, the highest weight is usually assigned to Bank 3, modeling spinning RSOs. Thus, the HME was able to not only detect the initial attitude profile of an RSO, but was able to detect when a change in the attitude profile occurred and correctly identify the new attitude profile of the RSO.

Spin Rate Change Detection Experiment

In another experiment, the RSO being tracked had a spinning attitude profile throughout the entire experiment, but halfway through the observation window, the spin rate about the major axis was changed from 0.1 rad/s to 0.7 rad/s. The "truth" RSO model used for this experiment was a hexagonal prism with 2m x 4m rectangular sides. For this experiment, the RSO was observed for 1 hour, with measurements available every 15 seconds. For the spin rate change experiment, the HME configuration is given in Table 7 and the top level and filter level gating weights are shown in Fig. 6.

Table 7. HME Configuration to detect a change in spin rate

Bank	Attitude Profile	Expert	Spin Rate
1	Major Axis Spin	1	(0,0,0.1) rad/s
		2	(0,0,0.5) rad/s
		3	(0,0,0.7) rad/s
2	Inertially Pointed	1	(0,0,0) rad/s

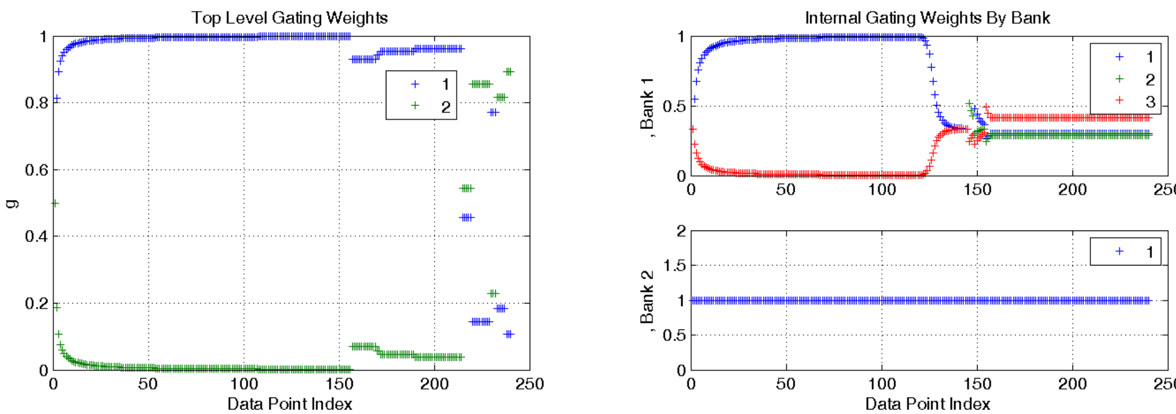

Fig. 6 Gating Weights for Spin Rate Change Experiment

Throughout most of the experiment, the highest top level gating weight is assigned to Bank 1, modeling major-axis spinning RSOs. The other bank in the experiment, Bank 2, was modeling an inertially-pointed RSO. The highest filter level weight is initially assigned to Filter 1, modeling an RSO with a 0.1 rad/s spin rate. After the change in the "truth" RSO's spin rate, the highest filter weight is then assigned to Filter 3, modeling an RSO with a 0.7 rad/s spin rate. Thus, the HME successfully detected a change in spin rate and determined the correct spin rate value.

6. CONCLUSIONS

Table 6 summarizes the results of our attitude profile detection experiments. Over a 1 hour observation window, with measurements every 15 seconds, the HME is able to assign the highest weight to the correct bank for a spinning or a sun-pointing object for 100% of time steps, and is usually able to distinguish a nadir-pointing object. These results show the HME is capable of distinguishing between three different RSO attitude profiles: nadir-pointing, sun-pointing, and major-axis spinning at a constant rate. When the truth RSO model is spinning, the HME was able to recognize that the RSO was not nadir-pointing or sun-pointing virtually all of the time. It was also able to identify that a sun-pointing object was not spinning or nadir-pointing essentially all of the time. For a nadir-pointing RSO, the HME is usually able to recognize that the object is not spinning or sun-pointing, although the results are not as clear cut as for the spinning or sun-pointing RSO. Identification of a nadir-pointing object is even more difficult when the object of interest displays some degree of symmetry (such as a cube or an axially symmetric hexagonal prism); although the HME is still usually able to distinguish that the RSO is nadir-pointing most of the time. The percentages given in Table 14 are the percentage of time steps (out of approximately 250) where the correct bank (that is, the bank with an attitude profile model matching the truth RSO model) was assigned the highest weight by the HME.

Table 6. Attitude Profile Detection Rate

RSO Attitude Profile	RSO Model		
	2m x 1m x 3m Cuboid	**Hexagonal Prism with 2m x 4m sides**	**2m x 2m x 2m Cube**
Spinning at $\omega = (0, 0, 0.3)$ rad/s	100% detection	100% detection	100% detection
Nadir-Pointing	85.4% detection	62.5% detection	57.9% detection
Sun-Pointing	100% detection	100% detection	100% detection

In addition, the HME is capable of detecting when changes occur in the attitude profile of an RSO. Our HME correctly identified when a nadir-pointing RSO began spinning about its major axis. Also, the HME was able to detect the correct spin rate of a major-axis spinning RSO both prior to and after a change in the spin rate.

We have shown that RSO attitude profile detection is possible despite the fact that the experts, all EKFs, in the HME only estimate position and velocity and do not estimate attitude directly. All information on RSO attitude obtained by the HME comes from the measurements processed in the experts, through the measurement residuals. While the HME does not estimate the attitude state of an RSO, the weights assigned to the various banks and experts in the HME indicate which attitude profile best matches the behavior of the observed RSO. While others have been able to determine attitude from brightness data, as in [9], it is significant that ***we could do so without running attitude estimation filters, which would require a higher processing rate than estimating translational motion alone***.

7. ACKNOWLEDGEMENTS

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